first known to Europeans, and doubtless for centuries before that time, were in a social stage at least as far advanced as that of our German ancestors in the days of Tacitus. We know that these barbarians, if we choose so to style them, had evolved a regular system of government, combining very ingeniously the methods of democracy and aristocracy, and comprising the germs of the English constitution. On this point the often-cited passage of Montesquieu will bear to be requoted and emphasized. "In perusing," writes the great legist, "the admirable treatise of Tacitus 'On the customs of the Germans,' we find it is from that nation the English have borrowed the idea of their political government. *This beautiful system was invented first in the woods.*" Will any one reply that the German barbarians, being of the Aryan stock, must be supposed capable of intellectual achievements which barbarians of the Indian race could not be expected to compass? I think the able and liberal-minded reviewer will agree with me, that reasoning of this 'high priori' sort, which assumes the very point in question, would be any thing but logical or satisfactory.

The reviewer is kind enough to say that many of the chapters in my volume "indicate immense research, and are of great value both ethnologically and philologically." I can assure him that equal diligence was exercised in preparing the chapters on the league and its founders, and I know of no reason why they should be deemed less accurate or less valuable. In these, moreover, as well as for the other portions of the work, I have been careful to indicate the sources of my information. Nothing will be easier than for any one who has doubts as to its correctness to repeat my inquiries, and to satisfy himself on that point. But I am happy to say that the communications which reach me from many quarters seem to show that no such doubts are likely to be entertained; at least, by any well-informed persons. Writers of the highest authority on American and Indian history receive the statements of the book as entirely authentic, and speak of it in terms too flattering for me to repeat.

Let me conclude by expressing the pleasure with which I have learned from this review that the valuable work of the excellent and indefatigable missionary-linguist, the late Father Marcoux, on the Iroquois language, is about to be published by the Bureau of ethnology. The idioms of the Huron-Iroquois group stand, perhaps, at the head of the best-known Indian languages as subjects of philosophical study. It is doubtful if even the Quichua or the Aztec equals them in comprehensive force, or in subtlety of distinctions. More than two centuries ago the learned missionary Brebeuf was struck with the resemblance of the Huron to the Greek; and in our own day Professor Max Müller, after a careful study of the Mohawk tongue, has expressed the opinion that the people who wrought out such a language 'were powerful reasoners and accurate classifiers.' The works of M. Marcoux, in conjunction with those of his distinguished pupil and successor, M. Cuoq, will afford ample means for the study of one, and perhaps the finest, of this remarkable group of languages.

In connection with this subject, it is proper to refer to the doubt expressed by the reviewer as to the correctness of the linguistic works of the French missionaries. It is suggested that they have made mistakes in grammar, and in particular that they have not been able to distinguish between the feminine and the indeterminate inflections. Now, it must be remembered that the intelligent and well-educated missionaries, whose competency is thus questioned, have for many years spoken and written the Iroquois language almost as familiarly as their native speech, and have published many books in that language for the use of their converts. Their predecessors, whose experience they have inherited, had been engaged in the same work for more than two hundred years. To suppose them so grossly ignorant of the grammar of the language as is now suggested is much the same as supposing a professor of Latin in an English or American college to be unable to distinguish between the genitive and the accusative cases in that language. If the work of Marcoux is so erroneous, it is clearly unfit to be published in a national series like that of the Ethnological bureau. In justice both to the missionaries and the bureau, I am glad to be able to show, by the best possible evidence, that the suspected errors do not exist. The Iroquois must be supposed to know their own language. The text of their Book of rites, fortunately, presents a test which is conclusive. In preparing the translation of this text, with the aid of the best native interpreters, I had occasion, as the appended glossary shows, to make constant use of the publications of M. Cuoq on the Iroquois tongue, and found them invariably correct. In particular, I may mention, the indeterminate form frequently occurs, employed precisely as indicated by him. The bureau may therefore safely add the work of M. Marcoux to the other valuable publications which have done so much credit to the scholarship of their authors and to the liberality of the government. H. HALE.

THOMSON AND TAIT'S NATURAL PHILOSOPHY. — I.

A treatise on natural philosophy. By Sir WILLIAM THOMSON LL.D., D.C.L., F.R.S., and P. G. TAIT, M.A. Vol. i., part ii., new edition. Cambridge, *University press*, 1883. 25+527 p. 8°.

THE first edition of vol. i. (23+727 p.) of this work was published by the delegates of the Clarendon press at Oxford, 1867. The authors then intended, as appears from their preface, to complete the work in four volumes. The remaining three volumes have, however, never appeared, much to the regret of all students of mathematical physics; and the authors state that the "intention of proceeding with the other volumes is now definitely abandoned."

In 1879 a new and enlarged edition was published of a portion of vol. i., entitled part i. (17+508 p.), including that part of the first edition contained in the first 336 pages; and now we have the remainder of vol. i., entitled part ii., which has been enlarged by important additions from 390 to 527 pages.

At p. 22 will be found a schedule of the alterations and additions in part i., and, at p. 24, those of part ii. "The most important part of the labor of editing part ii. has been borne by Mr. G. H. Darwin," whose remarkable papers in the Philosophical transactions upon the mathematical physics of the earth,

past and present, have placed him in the front rank of the cultivators of that science. His contributions to part ii. are duly accredited to him in the above-mentioned schedule.

The original object of this treatise is stated to be twofold; viz., "to give a tolerably complete account of what is now known of natural philosophy, in language adapted to the non-mathematical reader, and to furnish to those who have the privilege which high mathematical acquirements confer, a connected outline of the analytical processes by which the greater part of that knowledge has been extended into regions as yet unexplored by experiment."

From the nature of the case, the success of the authors in the attainment of their first object was small, compared with the second; for in order to give an intelligible account, to one unaccustomed to mathematical reasoning, of the general tenor and results of such reasoning, requires not only capacities such as few mathematicians have had in our day, except Clifford, but requires, also, an amount of space incompatible with the second and principal object which the authors had in view. In order, however, better to reach the non-mathematical reader, the authors published a work entitled 'Elements of natural philosophy, part i.,' which was only an abridgment of this 'treatise,' made by simply omitting all the advanced mathematical developments.

The second and principal object, however, of the authors, was one in which they, perhaps, were better fitted to succeed than any who could be selected. Their object was a large one, and its attainment was undertaken in a large way. It involved the presentation of the general subject of kinematics, or the geometry of motion considered apart from the forces causing it, including the exposition and use of generalized co-ordinates; and the consideration of harmonic motion, which "naturally leads to Fourier's theorem, one of the most important of all analytical results as regards usefulness in physical science," and including, also, the higher parts of the analytical discussion of curves and surfaces in space, of three dimensions. Next it required an extended development of dynamical laws and principles founded on Newton's Principia, comprising the dynamics of a particle and of a rigid body, and the whole of what is now termed kinetics worked over and "developed from the grand basis of the conservation of energy." The scope of the work demanded, also, the establishment of the principal formulae of spherical harmonics, a branch of analysis whose character we shall explain more at length hereafter.

All these and other subjects, which are usually regarded as but distantly related to the subject in hand, form a necessary part of a work whose object is as wide as that proposed by the authors. But it is hardly too much to say, that every important theory treated has received at their hands, not only elucidation, but additions of importance.

In order to make this paper as useful as may be, it has seemed best, in what follows, to content ourselves with the attempt to give an account to mathematical readers of the more important developments contained in the work, and not to engage in the task of trying to make an elucidation of its contents suitable for the general reader.

When we come to consider in particular the contents of part ii., it is found to be upon the general subject of *statics;* though many subjects, such as elasticity, the tides, etc., not usually treated in works on that subject, are here included. It consists of three chapters, the first of which is but five pages in length, and is merely introductory. It states and illustrates the utter impossibility of submitting the *exact* conditions of any physical question to mathematical investigation by reason of our ignorance of the nature of matter and molecular forces, but shows that approximate solutions obtained by neglecting forces which do not affect the conclusions sought to be established, and by regarding bodies as rigid which are nearly so, lead to practically the same results, as to the equilibrium and motion of bodies, as we should be led to by the solution of the infinitely more transcendent problem which has regard to *all* the forces acting.

In case, however, we consider the bending or other deformations of bodies regarded as elastic, we make a second approximation to the exact treatment of physical questions; and, by introducing modifications of elasticity due to changes of temperature, we should make a third approximation, which might be carried one step farther by taking account of conduction of heat, and farther still by considering the modifications of ordinary conduction due to thermo-electric currents, etc. In view of all this, the authors say, "The object of the present division of this volume (i.e., part ii.) is to deal with the first and second of these approximations. In it we shall suppose all solids either *rigid* (i.e., unchangeable in form and volume) or *elastic;* but, in the latter case, we shall assume the law connecting a compression or a distortion with the force which causes it, to have a particular form deduced from experiment. . . . We shall also suppose fluids,

whether liquids or gases, to be either *compressible* or *incompressible*, according to certain known laws; and we shall omit considerations of fluid friction, although we admit the consideration of friction between solids.''

The next chapter (v.) comprises pp. 6 to 100, and its especial object is set forth in the introductory section (454), as follows: "We naturally divide statics into two parts, — the equilibrium of a particle, and that of a rigid or elastic body or system of particles, whether solid or fluid. In a very few sections we shall dispose of the first of these parts, and the rest of this chapter will be devoted to a digression on the important subject of attraction." In other words, this chapter is devoted, with the exception of a couple of pages, to an extended treatment of attraction according to the law of the inverse square of the distance as applied to gravitation, electricity, and magnetism.

After a brief investigation of the usual formulae for the attraction of the spherical shell, circular disk, thin cylinder, circular arc, etc., the main subject of the chapter is reached, which is the modern mathematical theory of potential; which theory is the principal means now employed in the discussion of questions relating to the distribution of attracting matter, and the forces caused by it. This theory, due as it is to the analytical discoveries of Laplace, Green, Gauss, and others, might, nevertheless, have long remained comparatively barren of fruitful results in physics, had it not been for the genius of Faraday, who, though unskilled in the use of analysis, had a most powerful grasp of geometric and physical relations. In the words of another,[1] "Faraday, in his mind's eye, saw lines of force traversing all space, where mathematicians saw centres of force attracting at a distance; Faraday saw a medium where they saw nothing but a distance; Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they found it in a power of action at a distance." He conceived of lines of gravitational force as holding the planets in their orbits. These lines radiated through all space from the attracting body as a nucleus, regardless of the existence or non-existence of bodies upon which the attraction could be exerted. Furthermore, Faraday thought of each attracting body as surrounded at different distances by successive level surfaces, — like that of the ocean, for example, or the upper limit of the atmosphere; which surfaces cut the lines of force everywhere at right angles. This was not only true of gravitating matter, but each electrified body also had its system of lines of electrical force, and its corresponding system of level surfaces; and each magnet had its magnetic system as well. The geometry of these lines and surfaces is the basis of Faraday's reasoning in his 'Experimental researches,' and is the geometric truth hidden in the analytic discoveries clustering around Laplace's, Poisson's, and Green's theorems.

[1] Preface of Maxwell's Electricity and magnetism.

That we may call these relations more clearly before the mind, consider for a moment the so-called 'equation of continuity' of an incompressible fluid; which equation is divined from the geometric truth, that the quantity of such a fluid, which flows into any assumed closed surface, taken entirely within it, is equal to that flowing out, or that the total *flow* is *nil*. This is precisely expressed by the equation

$$\int F\,d\,S = 0, \qquad (1)$$

in which $d\,S$ is the area of the element of the assumed closed surface, F is the normal flow per square unit at that element, and the limits of integration are so taken that it extends over the entire surface. There is also another form of the equation of continuity, expressing the kinematic truth, that, in an incompressible fluid, the variations of the component velocities in the directions x, y, z, balance; i.e., their algebraic sum is *nil*, which may be written thus: —

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0, \qquad (2)$$

in which u, v, w, are the component velocities in the directions x, y, z, respectively.

Now, it is not difficult to picture to the mind the motions occurring within the mass of an incompressible fluid; such as water, for example. In whatever way it may be moving, we can think of stream-lines along which the different parts of it flow. A number of these lines, side by side, can be taken to form a stream, and can be thought of as bounded by a kind of tubular surface; which surface might be regarded as the boundary of the stream, which isolates it from surrounding streams. If the stream has the same velocity at every point along the tube, then its cross-section must be uniform; but, where the velocity is less, the cross-section is proportionately increased, and *vice versa*. This follows from the fact that the same quantity must pass each cross-section per unit of time. A tube in which a unit of volume passes a given cross-section per unit of time is called a *unit-tube*. Now, the forces of attraction in free space, caused by any distribution of matter, electricity, or magnetism,

follow precisely the same laws as the velocities and flow of incompressible fluids; for, consider for the moment the lines of force starting from the surface of some attracting body (a magnet, for example). They gradually diverge as the distance increases, and curve away into space. Each one of these lines may be taken as the representative of a definite amount of attraction, which is the same at all points along it; and if a tubular surface be supposed to exist, including everywhere certain of these lines which lie beside each other, and no others, the total amount of force acting across every cross-section of the tube is the same: hence equations (1) and (2) apply as well to forces of attraction as to velocities of an incompressible fluid, provided F, u, v, w, be taken to be the component forces along the normal and along x, y, z, respectively, and provided that none of the attracting matter be contained within the closed surface considered in equation (1), or at the point considered in equation (2). In order to the farther development of these equations, let us compute the work which would be obtained in carrying a unit of attracted material from one given position to another. The work is found from the usual expression

$$V = -\int(u\,dx + v\,dy + w\,dz), \tag{3}$$

in which u, v, w, being component forces, the limits of the integration are the co-ordinates of the two given points; but what path is taken between these points is of no consequence, because the amount of work depends alone upon their difference of level:

$$\therefore u = -\frac{dV}{dx}, v = -\frac{dV}{dy}, w = -\frac{dV}{dz}, \tag{4}$$

in which the right-hand numbers are partial differential coefficients. V is evidently a function of the co-ordinates such that its value depends upon position, and not upon the kind of co-ordinates employed. The point which fixes the lower limit of the integral in (3) is usually taken at infinity; and the value of V taken between it and the point fixing the upper limit is called the *potential* of the latter point.

By help of (3), we may put equation (1) in the form

$$\int\frac{dV}{du}\,dS = 0, \tag{5}$$

in which du is the element of the normal to the closed surface considered.

And by substituting in (2) the values given in (4), we have,

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0, \tag{6}$$

which is Laplace's equation, and is often written in the abbreviated form, $\nabla^2 V = 0$. Poisson showed, that, when the point at which the potential is to be computed is within the mass of the attracting matter, the right-hand member of (6) should no longer be *nil*, but $4\pi\rho$ instead, in which ρ is the density of the matter at that point. Similarly, the right-hand member of (5) becomes $4\pi m$ when an amount of matter m is included within the closed surface considered.

Equation (6) states that V must be such a function of the co-ordinates, that, if we take its three partial second differential coefficients and add them, their sum is *nil*. What possible algebraic forms are there which fulfil this condition? They are, of course, to be found by attempting to solve the differential equation (6). But it is to be seen beforehand, from the manner in which that equation was established, that it must have an infinite number of solutions; for V must be such a function as to be capable of expressing the work to be obtained from a unit of attracted matter when brought from infinity into the presence of attracting matter, whatever its distribution in space. The function V must therefore, in general, be different for every different distribution of attracting matter.

The integration of equation (6), and the discussion of its various solutions, constitute the branch of mathematics called spherical harmonic analysis; and to it the authors have devoted pp. 171 to 219, in part i. The formulae there obtained are employed, whenever required in the present chapter, to express the potential, or the attraction of matter distributed according to laws not conveniently to be treated by less elementary methods.

As the study of spherical harmonics has been comparatively neglected in this country, a short digression, explaining some of their properties, may be useful.

From the nature of attraction, it being toward fixed centres, it appears that polar co-ordinates would be more suitable to express its relations than rectangular co-ordinates; and, in fact, equation (6) is usually transformed to polar co-ordinates in space before integration, which co-ordinates may be taken to be the radius vector, the latitude, and the longitude of the point at which the potential is computed.

It may be shown that there are two general forms of solution of this polar differential equation, — one in ascending powers of the radius vector; and the other in ascending powers of its reciprocal, with coefficients depending upon sines or cosines of the angular co-ordinates.

As these series may be broken off at any point by the vanishing of the arbitrary numerical coefficients introduced during integration, these solutions may be in terms of the radius vector of any degree, positive or negative.

It is then found that a most important and simple class of solutions, called zonal harmonics, is those which are independent of the longitude, and consequently contain but two variables, — the radius vector and the latitude.

If in any harmonic we assume some special value of the radius vector for consideration, we evidently confine our attention to a spherical surface; and the expression is then spoken of as a surface harmonic, in distinction from that in which the radius vector is a variable, in which case it is called a solid harmonic.

On the surface of a sphere of given radius, it is possible to suppose the values of a surface-harmonic to be laid off graphically along the radii to each point, toward or away from the centre, according to their sign. This will give a picture to the mind of the distribution of the surface-harmonic.

Now, in a zonal harmonic of the first positive degree (which varies as the sine of the latitude) the surface-distribution is all positive on one side of the equator, and all negative on the other. A simple zonal harmonic of the second degree has a distribution like that included between a nearly spherical ellipsoid of revolution about the polar axis and a sphere when the two intersect along two parallels of latitude. The ellipsoid may be prolate or oblate. The number of zones depends, in any case, upon the degree of the zonal harmonic, and is such that the number of parallels of latitude at which the distribution changes sign is the same as the degree; and they are symmetrically situated about the equator, so that in the odd degrees the equator is itself such a parallel.

There are other solutions, called sectorial harmonics, in which the surface-distribution changes sign at equidistant meridians, and other solutions still, which are a combination of these two, called tesseral harmonics, in which the sign of the distribution changes, checkerboard fashion, at parallels and meridians. The sectorial harmonics are, however, in reality, nothing more than the combination of a number of zonal harmonics of the same degree, whose poles are situated at equal distances along the equator; and the tesseral harmonics are combinations of the sectorial with the zonal harmonics. Indeed, the most general harmonic is one by means of which any surface-distribution whatever may be expressed by properly determining the constant coefficients, and is merely a combination of zonal harmonics superposed one upon another, with poles situated in some irregular manner upon the surface of the sphere. This brings us to the fundamental theorem stated in section 537, upon which the special importance and usefulness of these functions rest, — "A spherical harmonic distribution of density (i.e., matter) on a spherical surface produces a similar and similarly placed spherical harmonic distribution of potential over every concentric spherical surface through space, external and internal; and so, also, consequently, of radial component force. . . . The potential is, of course, a solid harmonic through space, both external and internal; and is of positive degree in the internal, and of negative degree in the external space," as is evidently necessary, if the series expressing the potential in these two cases are to converge. When we come to treat in the same equation the potentials of a given point due to two different bodies, or systems of bodies, a remarkable relation is found to exist between them, called, from its discoverer, Green's theorem, which, though somewhat complicated when expressed in rectangular co-ordinates, has been put by Maxwell in a simple form, which may be written

$$\int V_1 dm_2 = \int V_2 dm_1, \qquad (7)$$

in which the subscripts refer to the first and second systems respectively, and the integrations are to be extended so as to include the total masses m_1 and m_2 respectively of the two systems. Laplace's and Poisson's equations are, of course, particular cases of Green's theorem. Thomson has effected an important extension of Green's theorem, given on pp. 167 to 171, part i. Constant references are made to these theorems, not only as to their direct application, as we have presented it, but in their application to the inverse question of determining what the distribution of matter must be to produce a given distribution of potential.

The most extended and important application of the theories of attraction and potential treated in this chapter is that of ellipsoids and ellipsoidal shells, — a subject which is closely connected with that of the figure of the earth, and one which has engaged the prolonged attention of many of the most powerful mathematical intellects of the past. A full account of the course of discovery in this field is found in Todhunter's History of the theories of attraction and figure of the earth, 2 vols.

Ten pages of new matter (pp. 40–50) have

been inserted in this edition, embracing modern investigations of importance on this subject.

(*To be continued.*)

OBLIGATIONS OF MATHEMATICS TO PHILOSOPHY, AND TO QUESTIONS OF COMMON LIFE.[1] — II.

I SAID that I would speak to you, not of the utility of the mathematics in any of the questions of common life or of physical science, but rather of the obligations of mathematics to these different subjects. The consideration which thus presents itself is, in a great measure, that of the history of the development of the different branches of mathematical science in connection with the older physical sciences, — astronomy and mechanics. The mathematical theory is, in the first instance, suggested by some question of common life or of physical science, is pursued and studied quite independently thereof, and perhaps, after a long interval, comes in contact with it, or with quite a different question. Geometry and algebra must, I think, be considered as each of them originating in connection with objects or questions of common life, — geometry, notwithstanding its name, hardly in the measurement of land, but rather from the contemplation of such forms as the straight line, the circle, the ball, the top (or sugar-loaf). The Greek geometers appropriated for the geometrical forms corresponding to the last two of these the words *σφαῖρα* and *κῶνος*, our sphere and cone; and they extended the word 'cone' to mean the complete figure obtained by producing the straight lines of the surface both ways indefinitely. And so algebra would seem to have arisen from the sort of easy puzzles in regard to numbers which may be made, either in the picturesque forms of the Bija-Ganita, with its maiden with the beautiful locks, and its swarms of bees amid the fragant blossoms, and the one queen-bee left humming around the lotus-flower; or in the more prosaic form in which a student has presented to him in a modern text-book a problem leading to a simple equation.

The Greek geometry may be regarded as beginning with Plato (B.C. 430–347). The notions of geometrical analysis, loci, and the conic sections, are attributed to him; and there are in his 'Dialogues' many very interesting allusions to mathematical questions, — in particular the passage in the 'Theaetetus' where he affirms the incommensurability of the sides of certain squares. But the earliest extant writings are those of Euclid (B.C. 285). There is hardly any thing in mathematics more beautiful than his wondrous fifth book; and he has also, in the seventh, eighth, ninth, and tenth books, fully and ably developed the first principles of the theory of numbers, including the theory of incommensurables. We have next Apollonius (about B.C. 247) and Archimedes (B.C. 287–212), both geometers of the highest merit, and the latter of them the founder of the science of statics (including therein hydrostatics). His dictum about the lever, his '*Εὕρηκα*,' and the story of the defence of Syracuse, are well known. Following these we have a worthy series of names, including the astronomers Hipparchus (B.C. 150) and Ptolemy (A.D. 125), and ending, say, with Pappus (A.D. 400), but continued by their Arabian commentators, and the Italian and other European geometers of the sixteenth century and later, who pursued the Greek geometry.

The Greek arithmetic was, from the want of a proper notation, singularly cumbrous and difficult; and it was, for astronomical purposes, superseded by the sexagesimal arithmetic, attributed to Ptolemy, but probably known before his time. The use of the present so-called Arabic figures became general among Arabian writers on arithmetic and astronomy about the middle of the tenth century, but it was not introduced into Europe until about two centuries later. Algebra, among the Greeks, is represented almost exclusively by the treatise of Diophantus (A.D. 150), — in fact, a work on the theory of numbers, containing questions relating to square and cube numbers, and other properties of numbers, with their solutions. This has no historical connection with the later algebra introduced into Italy from the east by Leonardi Bonacci of Pisa (A.D. 1202–1208), and successfully cultivated in the fifteenth and sixteenth centuries by Lucas Paciolus, or de Burgo, Tartaglia, Cardan, and Ferrari. Later on, we have Vieta (1540–1603), Harriot, already referred to, Wallis, and others.

Astronomy is, of course, intimately connected with geometry. The most simple facts of observation of the heavenly bodies can only be *stated* in geometrical language; for instance, that the stars describe circles about the Pole-star, or that the different positions of the sun among the fixed stars in the course of the year form a circle. For astronomical calculations it was found necessary to determine the arc of a circle by means of its chord. The notion is as old as Hipparchus, a work of whom is referred to as consisting of twelve books on the chords of circular arcs. We have (A.D. 125) Ptolemy's 'Almagest,' the first book of which contains a table of arcs and chords, with the method of construction; and among other theorems on the subject, he gives there the theorem, afterwards inserted in Euclid (book vi. prop. D), relating to the rectangle contained by the diagonals of a quadrilateral inscribed in a circle. The Arabians made the improvement of using, in place of the chord of an arc, the sine, or half-chord of double the arc, and so brought the theory into the form in which it is used in modern trigonometry. The before-mentioned theorem of Ptolemy, — or, rather, a particular case of it, — translated into the notation of sines, gives the expression for the sine of the sum of two arcs in terms of the sines and cosines of the component arcs, and it is thus the fundamental theorem on the subject. We have in the fifteenth and sixteenth centuries a series of mathematicians, who, with wonderful enthusiasm and perseverance, calculated tables of the trigonometrical or circular functions, — Purbach, Müller or Regiomontanus,

[1] Address of Professor CAYLEY before the British association. Concluded from No. 35.